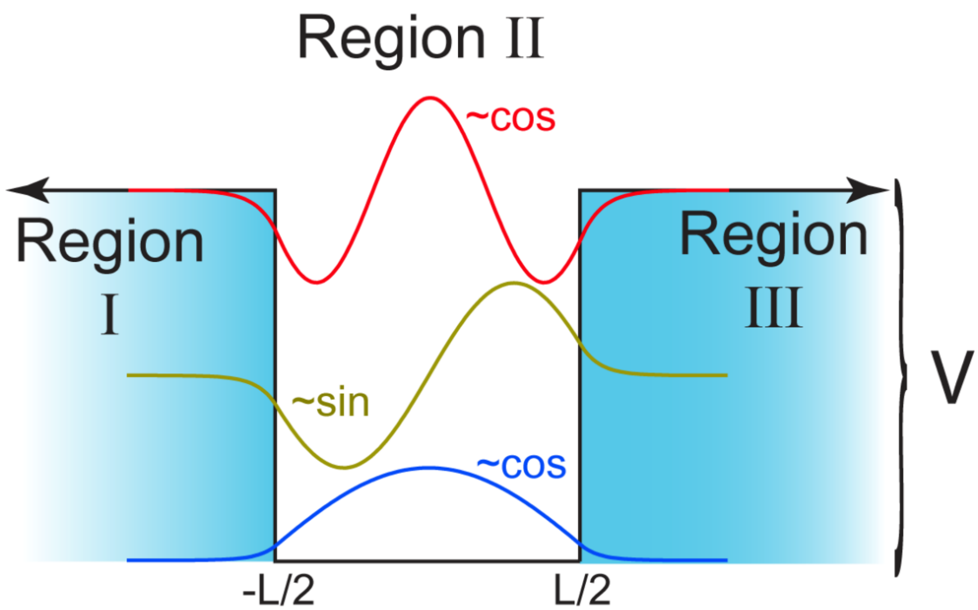

The centered box has wavefunctions that vary between cosine (the ground state and subsequent higher states) and sine, and are in a region with no potential energy. Thus, they have the form (for example), $\psi_{II} = A \cdot cos(k_2 \cdot x)$, where $k_2 = \sqrt{2mE}/\hbar$. The wavefunction in Region III is $\psi_{III} = B \cdot exp(i \cdot k_1 \cdot x)$, however, since we are only interested in wavefunctions with energies less than V, $k_1 = i\sqrt{2m(V-E)}/\hbar$. As a result, $\psi_{III} = B \cdot exp(-k_1 \cdot x)$.

Note that Region I is basically identical to Region III, and thus $\psi_I = B \cdot exp(k_1 \cdot x)$, where the argument of the exponential is positive because the x's are negative in Region I.

For the ground ~cosine state, starting with the stipulation of smooth and continuous:

Region II&III: Wavefunctions are equal: $A \cdot cos\left(k_2\left(\frac{L}{2}\right)\right) = B \cdot exp\left(-k_1\left(\frac{L}{2}\right)\right)$

Region II&III: Derivatives are equal: $-A \cdot k_2 \cdot sin\left(k_2\left(\frac{L}{2}\right)\right) = -B \cdot k_1 \cdot exp\left(-k_1\left(\frac{L}{2}\right)\right)$

Divide the two: $\frac{A \cdot cos\left(k_2\left(\frac{L}{2}\right)\right)}{-A \cdot k_2 \cdot sin\left(k_2\left(\frac{L}{2}\right)\right)} = \frac{B \cdot exp\left(-k_1\left(\frac{L}{2}\right)\right)}{-B \cdot k_1 \cdot exp\left(-k_1\left(\frac{L}{2}\right)\right)}$

Therefore: $\frac{-1}{k_2} cotan\left(k_2\left(\frac{L}{2}\right)\right) = \frac{1}{k_1}$; insert the definitions of k's:

$$cotan\left(\frac{\sqrt{2mE}}{\hbar}\left(\frac{L}{2}\right)\right) = \frac{\sqrt{2mE}/\hbar}{\sqrt{2m(V-E)}/\hbar} = \frac{\sqrt{E}}{\sqrt{(V-E)}} \qquad \textbf{(1)}$$

For the first excited ~sine state(s), starting with the stipulation of smooth and continuous:

Region II&III: Wavefunctions are equal: $A \cdot sin\left(k_2\left(\frac{L}{2}\right)\right) = B \cdot exp\left(-k_1\left(\frac{L}{2}\right)\right)$

Region II&III: Derivatives are equal: $A \cdot k_2 \cdot cos\left(k_2\left(\frac{L}{2}\right)\right) = -B \cdot k_1 \cdot exp\left(-k_1\left(\frac{L}{2}\right)\right)$

Divide the two: $\frac{A \cdot sin\left(k_2\left(\frac{L}{2}\right)\right)}{A \cdot k_2 \cdot cos\left(k_2\left(\frac{L}{2}\right)\right)} = \frac{B \cdot exp\left(-k_1\left(\frac{L}{2}\right)\right)}{-B \cdot k_1 \cdot exp\left(-k_1\left(\frac{L}{2}\right)\right)}$

Therefore: $\frac{1}{k_2} tan\left(k_2\left(\frac{L}{2}\right)\right) = \frac{-1}{k_1}$; insert the definitions of k's:

$$tan\left(\frac{\sqrt{2mE}}{\hbar}\left(\frac{L}{2}\right)\right) = \frac{-\sqrt{2mE}/\hbar}{\sqrt{2m(V-E)}/\hbar} = \frac{-\sqrt{E}}{\sqrt{(V-E)}} \qquad \textbf{(2)}$$

The same operations can be performed between Regions I&II, but these lead to the exact same results in terms of eqs. (1) & (2).

Now we can define energy by finding the E that satisfies equations (1) and (2), which has to be done using some computer program like Matlab or Origin.