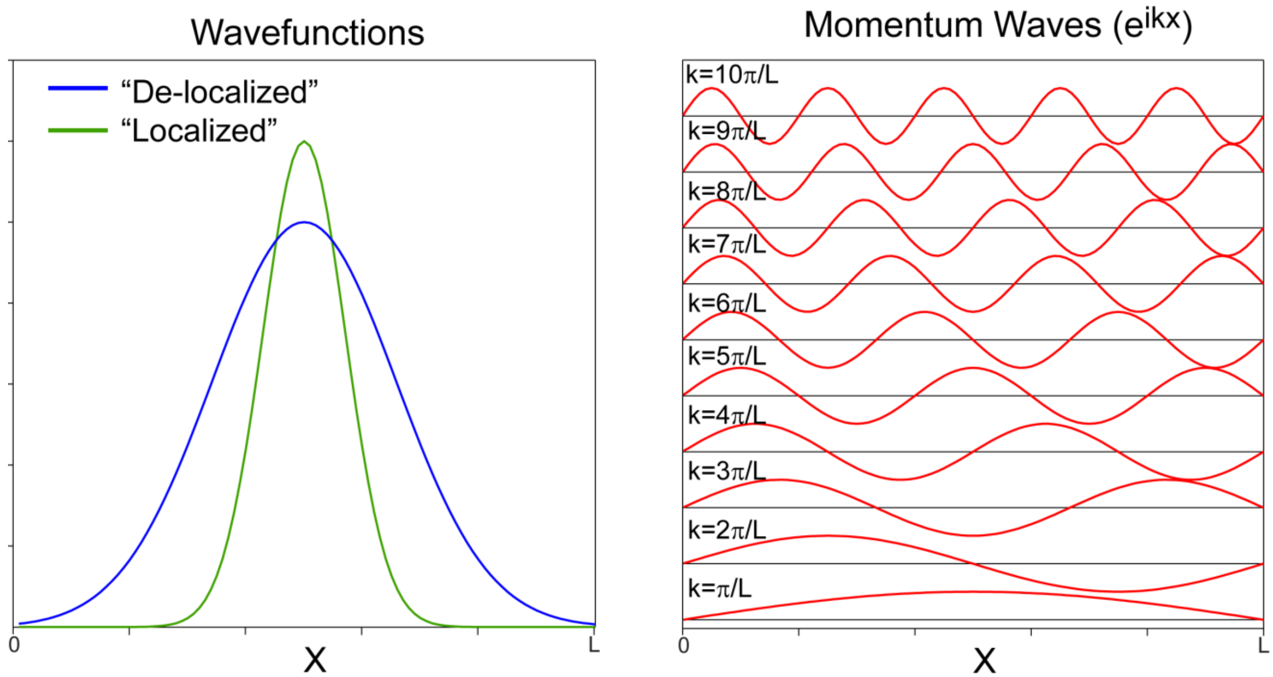
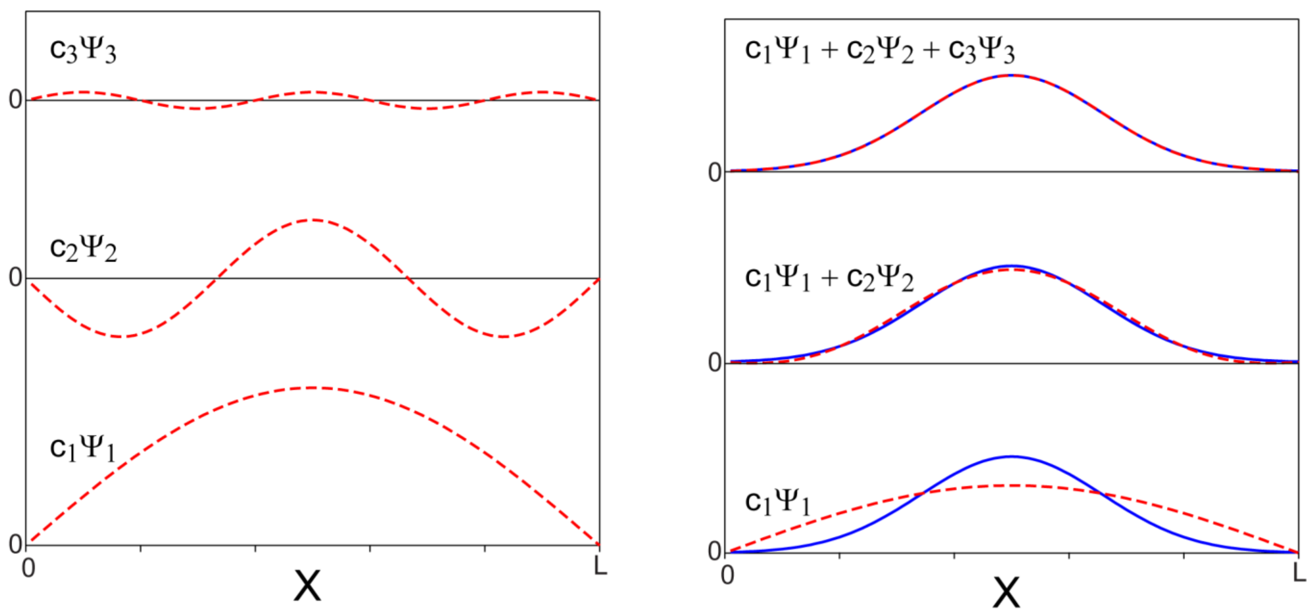
Uncertainty in Position and Momentum

For this handout, we are going to show how the localized and delocalized wavefunctions in the box below (left) can be decomposed into the momentum waves on the right.



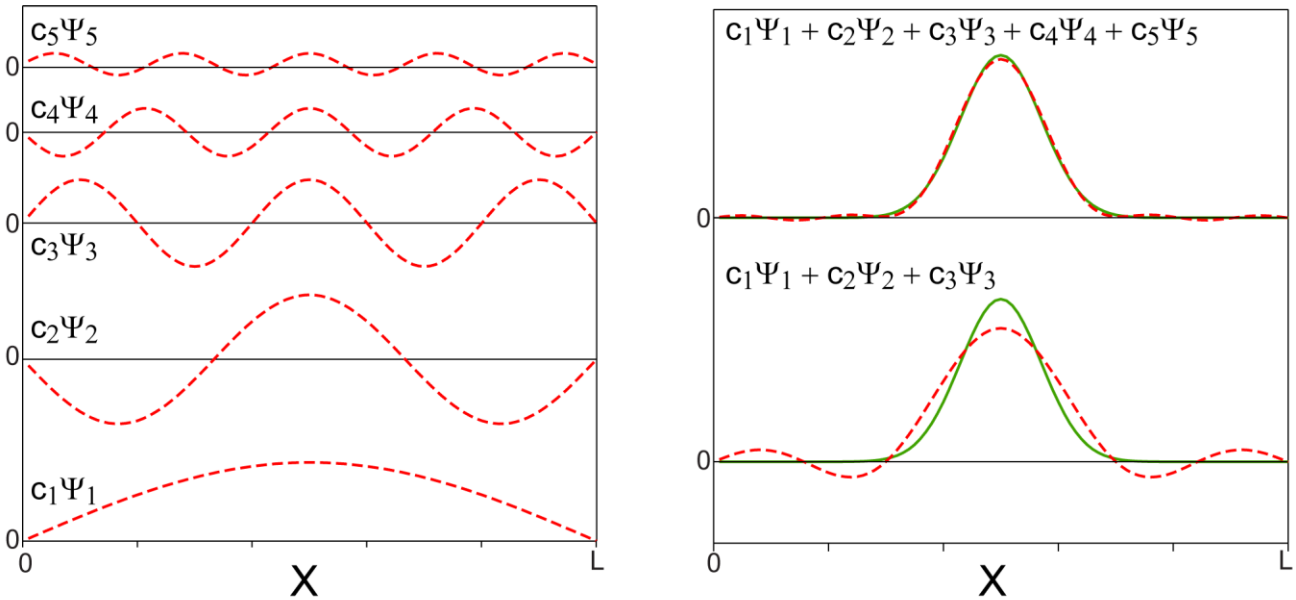
Delocalized Ψ: To show how we can equate the delocalized wavefunction as linear combination of a few momentum waves, on the left are three waves that are weighted by "c" factors. On the right are the waves added together in comparison to the original state. As you can see, adding two momentum waves is pretty good, and the sum of all three looks almost identical to the original wavefunction. Addition of more momentum waves would help, but isn't necessary.



Localized Ψ: On the left are five weighted momentum waves, which are summed to equate to the original more localized wavefunction on the right. As can be seen, the addition of three waves isn't a good match. Rather, it takes a total of five waves:

$\Psi = c_1 \times wave_1 + c_2 \times wave_2 + c_3 \times wave_3 + c_4 \times wave_4 + c_5 \times wave_5$

before the linear combination looks good.



The point is that the wide delocalized wavefunction is mostly composed of 3 momentum waves, meaning that the momentum is relatively well-known. However, the narrower, localized wavefunction has 5 momentum waves. As a result, determining the average momentum will require more measurements, which is why the momentum is less certain.

To put a more mathematical representation, the total momentum of the delocalized Ψ is:

$\langle p \rangle \approx |c_1|^2 \times k_1 + |c_2|^2 \times k_2 + |c_3|^2 \times k_3$

where the k's are the momentum of the waves which are measurable because those states are eigenfunctions of $\hat{p}$, the momentum operator. Any measurement of momentum on Ψ may return one of three values ($k_1 \ldots k_3$); the probability of measuring any particular one it equal to $|c_k|^2$.

For the localized state, $\langle p \rangle \approx |c_1|^2 \times k_1 + |c_2|^2 \times k_2 + |c_3|^2 \times k_3 + |c_4|^2 \times k_4 + |c_5|^2 \times k_5$. Any measurement of momentum may return one of five values ($k_1 \ldots k_5$), the probability of measuring any particular one it equal to $|c_k|^2$. Again, we see how there is less certainty in the result in the localized state.