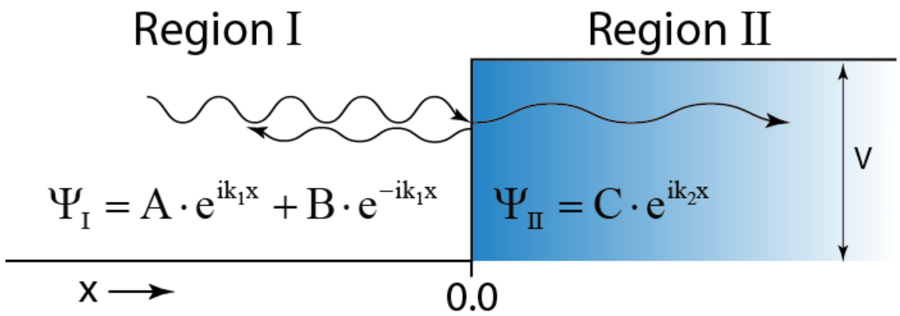
Reflection Across a Wall



Region I

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = E \cdot \Psi$$

$\Psi_I(x) = Ae^{ik_1x} + Be^{-ik_1x}$

Region II

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = E \cdot \Psi$$

$\Psi_{II}(x) = Ce^{ik_2x}$

IF E<V:

If $\frac{\partial^2\Psi}{\partial x^2} = -k_1^2 \cdot \Psi$ then $k_1 = \sqrt{2mE}/\hbar$

If $\frac{\partial^2\Psi}{\partial x^2} = -k_2^2 \cdot \Psi$ then $ik_2 = i\sqrt{2m(V-E)}/\hbar$

Region I

$\Psi_I(x) = Ae^{ik_1x} + Be^{-ik_1x}$

$$\frac{\partial\Psi}{\partial x} = ik_1Ae^{ik_1x} - ik_1Be^{-ik_1x}$$

Region II

$\Psi_{II}(x) = Ce^{i \cdot ik_2x} = Ce^{-k_2x}$

$$\frac{\partial\Psi}{\partial x} = -k_2Ce^{-k_2x}$$

Continuous at x= 0:

$Ae^0 + Be^0 = Ce^0$ therefore: $A + B = C$ (1)

Smooth at x=0:

$ik_1A - ik_1B = -k_2C$ (2)

Solution to $\frac{B}{A}$: Start with (1) to derive C = A + B and plug this into (2):

$ik_1A - ik_1B = -k_2(A + B)$

$ik_1A + k_2A = -k_2B + ik_1B$ therefore: $A(ik_1 + k_2) = B(-k_2 + ik_1)$

$\frac{B}{A} = \frac{ik_1+k_2}{ik_1-k_2}$. You can make this look simpler by multiplying by $\frac{-i}{-i}$: $\frac{B}{A} = \frac{k_1-ik_2}{k_1+ik_2}$

Note that **%Reflectance** is: $\frac{|B|^2}{|A|^2} = \frac{(ik_1+k_2)^*}{(ik_1-k_2)^*} \times \frac{ik_1+k_2}{ik_1-k_2} = \frac{-ik_1+k_2}{-ik_1-k_2} \times \frac{ik_1+k_2}{ik_1-k_2} =$

$\frac{k_1^2+k_2^2-ik_1k_2+k_2ik_1}{k_1^2+k_2^2+ik_1k_2-k_2ik_1} = \frac{k_1^2+k_2^2}{k_1^2+k_2^2} = 1.$ The **%Transmittance** is $1 - \frac{|B|^2}{|A|^2} = 0$

IF E>V:

Region I

$k_1 = \sqrt{2mE}/\hbar$

$\Psi_I(x) = Ae^{ik_1x} + Be^{-ik_1x}$

$\frac{\partial\Psi}{\partial x} = ik_1Ae^{ik_1x} - ik_1Be^{-ik_1x}$

Region II

$k_2 = \sqrt{2m(E-V)}/\hbar$

$\Psi_{II}(x) = Ce^{ik_2x}$

$\frac{\partial\Psi}{\partial x} = ik_2Ce^{ik_2x}$

Continuous at x= 0:

$Ae^0 + Be^0 = Ce^0$ therefore: $A + B = C$ (1)

Smooth at x=0:

$ik_1A - ik_1B = ik_2C$ (2)

Solution to $\frac{B}{A}$: Start with (1) to derive C = A + B and plug this into (2):

$ik_1A - ik_1B = ik_2(A + B)$

$ik_2B + ik_1B = ik_1A - ik_2A$ therefore: $iB(k_2 + k_1) = iA(k_1 - k_2)$ and thus:

$\frac{B}{A} = \frac{k_1-k_2}{k_1+k_2}$. Note that

%Reflectance is: $\frac{|B|^2}{|A|^2} =$

$\frac{k_1^2+k_2^2-2k_1k_2}{k_1^2+k_2^2+2k_1k_2}$

The **%Transmittance** is:

$1 - \frac{|B|^2}{|A|^2} = \frac{4k_1k_2}{k_1^2+k_2^2+2k_1k_2}$.