Engineering Maxwell-Boltzmann Problems

Here is an example on how a Maxwell-Boltzmann type question can be prepared.

1) First, the form of the distribution must be determined. Here is an example of a skewed bell-shaped curve, which is a function of pigs: $\mathrm{pig} \cdot \mathrm{e}^{-\mathrm{pig}^2} \cdot \partial\mathrm{pig}$.

Now, the distribution must follow certain rules. For example, the argument of the exponential cannot have units. Thus, we have to modify it as so: $\mathrm{e}^{-\mathrm{pig}^2/\mathrm{b}^2}$, where "b" are blankets and have the same units as pigs. Likewise, $\mathrm{pig} \cdot \partial\mathrm{pig}$ has units of pigs^2, thus we have to divide by b^2:

$$\frac{\mathrm{pig}}{\mathrm{b}^2} \cdot \mathrm{e}^{-\mathrm{pig}^2/\mathrm{b}^2} \cdot \partial\mathrm{pig}$$

While it would appear that we are done, actually we need an additional term in the exponential to tune the average value. We will call this the "c" constant. Likewise, the expression must be normalized, which is accomplished by multiplying the expression by the "N" constant:

$$\mathrm{N} \cdot \frac{\mathrm{pig}}{\mathrm{b}^2} \cdot \mathrm{e}^{-\mathrm{c}\cdot\mathrm{pig}^2/\mathrm{b}^2} \cdot \partial\mathrm{pig}$$

Now here is the engineering part because we have one equation with two unknowns (N and c).

1) Normalization. The expression must be normalized:

$$\int_0^\infty \mathrm{N} \cdot \frac{\mathrm{pig}}{\mathrm{b}^2} \cdot \mathrm{e}^{-\mathrm{c}\cdot\mathrm{pig}^2/\mathrm{b}^2} \cdot \partial\mathrm{pig} = \frac{\mathrm{N}}{\mathrm{b}^2} \int_0^\infty \mathrm{pig} \cdot \mathrm{e}^{-\mathrm{c}\cdot\mathrm{pig}^2/\mathrm{b}^2} \cdot \partial\mathrm{pig} = 1$$

Whenever confronted with calculus, we Google an identity. Here we find: $\int_0^\infty \mathrm{x} \cdot \mathrm{e}^{-\mathrm{a}\cdot\mathrm{x}^2} \cdot \partial\mathrm{x} = \frac{1}{2\mathrm{a}}$.

When applied to our expression we see some constants that can come out of the expression and $\mathrm{a} = \frac{\mathrm{c}}{\mathrm{b}^2}$:

$$\frac{\mathrm{N}}{\mathrm{b}^2} \int_0^\infty \mathrm{pig} \cdot \mathrm{e}^{-\mathrm{c}\cdot\mathrm{pig}^2/\mathrm{b}^2} \cdot \partial\mathrm{pig} = \frac{\mathrm{N}}{\mathrm{b}^2} \frac{\mathrm{b}^2}{2 \cdot \mathrm{c}} = 1$$

Therefore $\mathrm{N} = 2 \cdot \mathrm{c}$, and the distribution is now: $\frac{2\cdot\mathrm{c}}{\mathrm{b}^2} \cdot \mathrm{pig} \cdot \mathrm{e}^{-\mathrm{c}\cdot\mathrm{pig}^2/\mathrm{b}^2} \cdot \partial\mathrm{pig}$

2) Average value. I will assign an average value to the equation, here $\langle\mathrm{pigs}\rangle = 10 \cdot \mathrm{b}$. Note that you may have expected the average value to be $10 \cdot \mathrm{pigs}$. This actually isn't sensible- if you're not sure, ask yourself this- is the average velocity of a gas equal to 10 velocities? See, nonsense. Here, the average number of pigs is 10 blankets ("b"), which is consistent with the fact that pigs and blankets have the same units.

Therefore $\int_0^\infty \frac{2\cdot\mathrm{c}}{\mathrm{b}^2} \cdot \mathrm{pig}^2 \cdot \mathrm{e}^{-\mathrm{c}\cdot\mathrm{pig}^2/\mathrm{b}^2} \cdot \partial\mathrm{pig} = 10 \cdot \mathrm{b}$

Again, confronted with calculus we look up an expression: $\int_0^\infty x^2 \cdot e^{-a\cdot x^2} \cdot \partial x = \frac{\sqrt{\pi}}{4\cdot a^{3/2}}$.

Therefore $\langle \text{pigs} \rangle = \int_0^\infty \frac{2\cdot c}{b^2} \cdot \text{pig}^2 \cdot e^{-c\cdot \text{pig}^2/b^2} \cdot \partial\text{pig} = \frac{2\cdot c}{b^2}\frac{\sqrt{\pi}\cdot(b^2)^{3/2}}{4\cdot c^{3/2}} = \frac{2\sqrt{\pi}}{4}\frac{c}{c^{3/2}}\frac{b^3}{b^2} = \frac{2\sqrt{\pi}}{4}\frac{c}{c^{3/2}}\frac{b^3}{b^2} = \frac{\sqrt{\pi}\cdot b}{2\sqrt{c}}$

Recall the point is that this is equal to $10 \cdot b$, and we are solving for c. Therefore, we solve the following:

$$\frac{\sqrt{\pi}\cdot b}{2\sqrt{c}} = 10 \cdot b$$

which is simplified to : $\frac{\sqrt{\pi}\cdot b}{20b} = \sqrt{c}$ and thus $c = \frac{\pi}{400}$.

Since $N = 2 \cdot c = \frac{2\pi}{400}$, the final distribution is:

$$N \cdot \frac{\text{pig}}{b^2} \cdot e^{-c\cdot \text{pig}^2/b^2} \cdot \partial\text{pig} = \frac{2\pi}{400} \cdot \frac{\text{pig}}{b^2} \cdot e^{-\pi\cdot \text{pig}^2/400\cdot b^2} \cdot \partial\text{pig}$$

You can check the equation yourself or turn to the next page!

1) Check on normalization:

$$\int_0^\infty \frac{2\pi}{400} \cdot \frac{\mathrm{pig}}{\mathrm{b}^2} \cdot \mathrm{e}^{-\pi \cdot \mathrm{pig}^2 / 400 \cdot \mathrm{b}^2} \cdot \partial \mathrm{pig} = \frac{2\pi}{400 \cdot \mathrm{b}^2} \frac{400 \cdot \mathrm{b}^2}{2\pi} = 1$$

2) Check the average:

$$\int_0^\infty \frac{2\pi}{400} \cdot \frac{\mathrm{pig}^2}{\mathrm{b}^2} \cdot \mathrm{e}^{-\pi \cdot \mathrm{pig}^2 / 400 \cdot \mathrm{b}^2} \cdot \partial \mathrm{pig} = \frac{2\pi}{400 \cdot \mathrm{b}^2} \frac{\sqrt{\pi} \cdot (400)^{3/2} \cdot (\mathrm{b}^2)^{3/2}}{4 \cdot \pi^{3/2}} = \frac{2 \cdot 20^3 \cdot \pi\sqrt{\pi} \cdot \mathrm{b}^3}{4 \cdot 400 \cdot \pi^{3/2} \cdot \mathrm{b}^2} = 10 \cdot \mathrm{b}$$

3) Check the cat: